Quantum mechanical representation of acoustic streaming and acoustic radiation pressure

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We discuss acoustic streaming and acoustic radiation pressure from the viewpoint of energy and momentum of acoustic waves, using a quantum mechanical representation of acoustic waves. We represent the energy ε and momentum μ of acoustic waves as $\varepsilon = n_p \hbar \omega$ and $\mu = n_p \hbar k$; here n_p is the phonon density, ω is the frequency, k is the wave number, and $\hbar$ is Planck's constant. It is easy to derive the momentum of acoustic waves as $\mu = \varepsilon / c$ (c is the sound velocity). Therefore, we can represent the acoustic streaming and acoustic radiation pressure in terms of the momentum.

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I. INTRODUCTION

Acoustic waves are treated as phonons in the investigation of liquid helium 4 [1,2] and solids [3–5]. We have attempted to investigate the nonlinear phenomena (subharmonics [6], capillary waves and cavitation [7]) of acoustic waves in water, using the phonon concept [6–10] and quantum mechanics [6–8]. We treated acoustic waves from the viewpoint of energy and momentum [6–10] using the ω-k diagram [11,12].

Acoustic waves in water have been analyzed using hydrodynamics [13–16]. However, it is rather difficult to derive the momentum of an acoustic wave using hydrodynamics [17]. On the other hand, a quantum mechanical representation is very simple and clear; thus the derivation of the momentum of an acoustic wave becomes very easy [6–10]. Therefore, derivations of acoustic radiation pressure and the driving force of acoustic streaming are simple and clear.

We propose a quantum mechanical representation of acoustic waves. Using the momentum of an acoustic wave, we derive the driving force of acoustic streaming and acoustic radiation pressure. In this study, we try to distinguish clearly between pressure and force and thermal energy and kinetic energy from the viewpoint of momentum and energy.

II. THEORY

A. Quantum mechanical representation of an acoustic wave

A quantum mechanical representation of acoustic waves has been proposed [6–10]. Using the phonon density n_p, the energy density ε and momentum density μ are represented as,

$$\varepsilon = n_p \hbar \omega, \tag{1}$$

$$\mu = n_p \hbar k. \tag{2}$$

Here $\omega = 2\pi f$ (f is the frequency), $k = 2\pi/\lambda$ (λ is the wavelength) is the wave number, and $\hbar = h/2\pi$ (h is Planck's constant). Using Eqs. (1) and (2), we can represent the momentum as

$$\mu = \varepsilon / c, \tag{3}$$

where $c = \omega / k$ is the phase velocity of an acoustic wave. The derivation of Eq. (3) is rather difficult from an analysis of hydrodynamics [17].

B. Derivation of acoustic pressure and streaming force

Acoustic radiation pressure and acoustic streaming are due to the momentum and energy transfer of an acoustic wave to the medium. The difference between acoustic radiation pressure and acoustic streaming is that for acoustic streaming there is a transfer of acoustic energy to the kinetic energy of the medium. In the case of acoustic radiation pressure, the medium does not obtain kinetic energy. Although there is acoustic energy transfer to the thermal energy of the medium, there is no momentum transfer. This is because the momentum of thermal energy is 0. However, kinetic energy is related to momentum.

Figure 1 shows the conceptual illustration of acoustic en-

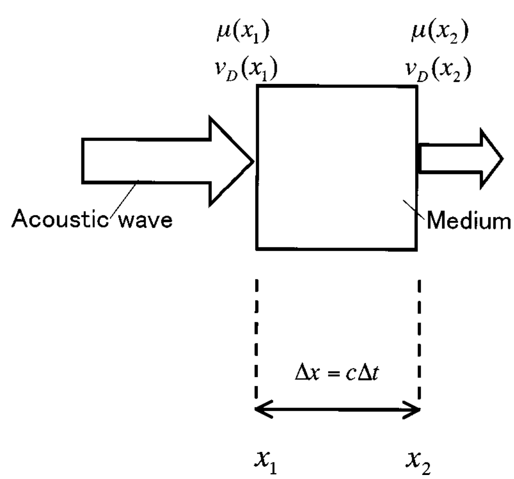

FIG. 1. Conceptual illustration of acoustic wave absorption.

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ergy and momentum transfer to the medium. Here we discuss ideal kinetic energy transfer to the medium. Phonons travel a distance $\Delta x = x_2 - x_1$ within time $\Delta t = t_2 - t_1$ with a phase velocity c; i.e. $c = \Delta x/\Delta t$, and the momentum of a phonon is transferred to the medium, which is represented as $\Delta\mu = \mu(x_2) - \mu(x_1)$. Thereafter the medium is accelerated, $\Delta\nu_D = \nu_D(x_2) - \nu_D(x_1)$; here ν_D shows the drift velocity of the medium. By assuming that the momentum is completely transferred to the medium drift motion, we can obtain Eq. (4), which indicates the driving force of the medium:

$$\rho \frac{\Delta\nu_D}{\Delta t} = -\frac{\Delta\mu}{\Delta t}, \tag{4}$$

Here we can substitute $\Delta t = \Delta x/c$ and $\mu = \varepsilon/c$, and then obtain

$$\rho \frac{\Delta\nu_D}{\Delta t} = -\frac{\Delta\mu}{\Delta t} = -\frac{\Delta\varepsilon}{\Delta x}. \tag{5}$$

Equation (5) shows that $\Delta\varepsilon = \varepsilon(x_2) - \varepsilon(x_1)$, which is the acoustic energy difference in the distance Δx, causes the driving force of the medium. Therefore, the driving force of acoustic streaming f_A is represented as follows [13] as the limit of $\Delta t \to 0$:

$$f_A \equiv \rho \frac{\partial\nu_D}{\partial t} = -\frac{\partial\varepsilon}{\partial x}. \tag{6}$$

Under the ideal condition that the medium does not move, the momentum transferred to the medium acts as the pressure of acoustic radiation. Under acoustic radiation pressure, the medium obtains momentum; however, does not gain kinetic energy.

We define the acoustic radiation pressure p_A as the average momentum transfer, as follows:

$$p_A \equiv -\frac{\Delta\mu}{\Delta t}. \tag{7}$$

In Eq. (7) there is no kinetic energy transfer to the medium; there is only momentum transfer. Equation (7) is identical to Eq. (4), and hence can be rewritten as follows:

$$p_A \equiv -\frac{\Delta\mu}{\Delta t} = -\frac{\Delta\varepsilon}{\Delta x}. \tag{8}$$

Here, the energy difference is caused through the reflection or absorption of an acoustic wave. Absorption means acoustic energy transfer to the thermal energy of the medium. Thermal energy transfer will be discussed in Secs. II C and II D.

The force of acoustic streaming f_A moves the medium; however, the pressure of the acoustic radiation p_A acts as the pressure on the medium. The phenomenon which occurs in the actual medium is a mixture of f_A and p_A acting on the medium (see Sec. III). Equations (6) and (8) are represented using the acoustic energy density; therefore, acoustic wave absorption and reflection are included in these expressions, i.e., Eqs. (6) and (8) represent not only the absorption but also the reflection of acoustic wave.

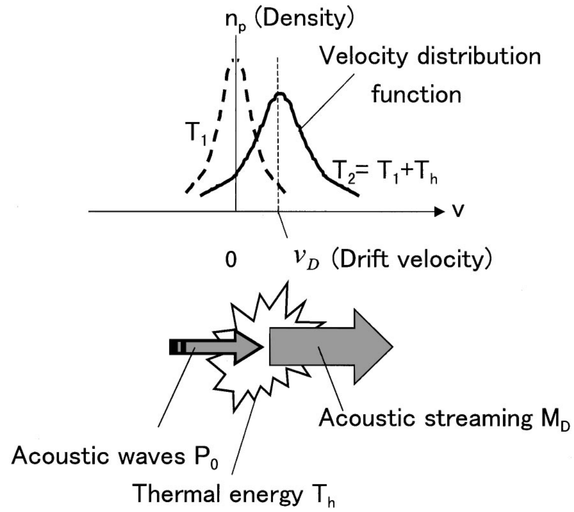

FIG. 2. Acoustic energy absorption by the medium.

C. Mechanism

The mechanisms of acoustic wave absorption are explained briefly. Figure 2 shows acoustic energy transfer to the thermal energy T_h and the medium motion M_D. Acoustic waves have not only energy but momentum; thus, at the absorption of acoustic waves, momentum should also be transferred to the medium, and acoustic streaming is generated.

At the Maxwell distribution, the temperature of the medium is represented by the width of the velocity distribution function of the medium. The thermal energy T_h is the energy absorbed in the medium as the width of the velocity distribution function of the medium. Therefore, there is no momentum transfer.

The medium motion M_D is represented by momentum $\rho\nu_D$ and a kinetic energy $\frac{1}{2}\rho\nu_D^2$; here ν_D is the drift velocity of the medium. The medium obtains thermal energy by the absorption of acoustic waves, which changes the thermal energy of the medium from T_1 to $T_2 = T_1 + T_h$. In the case in which the medium does not move, acoustic radiation pressure will be generated.

D. *ω*-*k* diagram

Energy and momentum conservation between an acoustic wave and the medium is represented on the frequency vs wave number (ω-k) diagram [11,12] shown in Fig. 3. Here the vertical axis indicates the frequency ω, and the horizontal axis indicates the wave number k. We represent the acoustic wave and medium motion in the ω-k diagram, which indicates energy and momentum transfer between the acoustic wave and the medium. Acoustic waves and medium motions are represented by the points in the diagram. Here the straight line O-a shows the dispersion relations of longitudinal waves. The quadratic curve m-O-m' is plotted by momentum $\rho\nu_D$ and kinetic energy $\frac{1}{2}\rho\nu_D^2$, using ν_D as a vari-

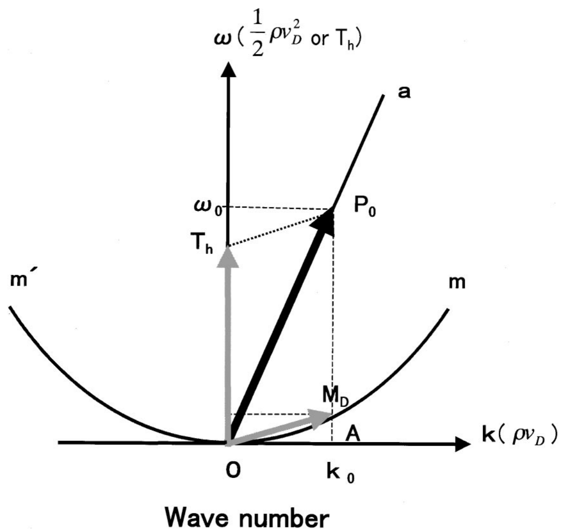

FIG. 3. Acoustic wave transfer to thermal energy and medium motion.

able. This curve represents the kinetic motion of the medium. The ω axis represents the kinetic energy of the medium $\frac{1}{2}\rho\nu_D^2$, and the k axis represents the momentum of the medium $\rho\nu_D$. Vector OP_0 is an acoustic wave which has a frequency ω_0 and a wave number k_0,OT_h is the thermal energy, and OM_D represents the kinetic motion of the medium drift. The thermal energy OT_h does not have momentum; thus it is represented by a vector on the ω axis. Parallelogram $OM_DP_0T_h$ shows the condition of energy and momentum conservation between the acoustic wave and the medium. This condition is a sufficient condition which indicates that the acoustic wave is transferred to the thermal energy and medium drift.

Figure 4 shows the ω-k diagram of acoustic radiation pressure that is generated by acoustic wave absorption. In this case, the medium does not obtain kinetic energy, but only gains thermal energy and acoustic radiation pressure *OA*. Here, parallelogram OAP_0T_h shows the energy and momentum conservation condition.

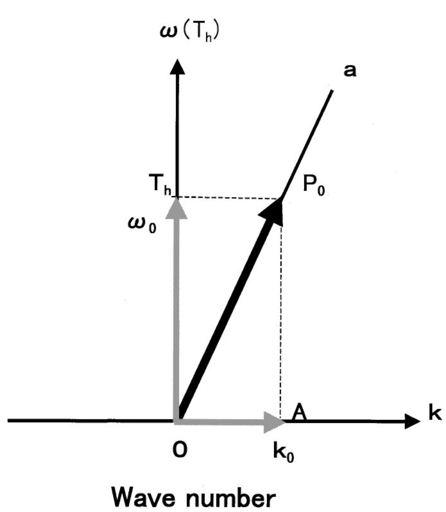

FIG. 4. ω-k diagram of acoustic wave absorption and radiation pressure.

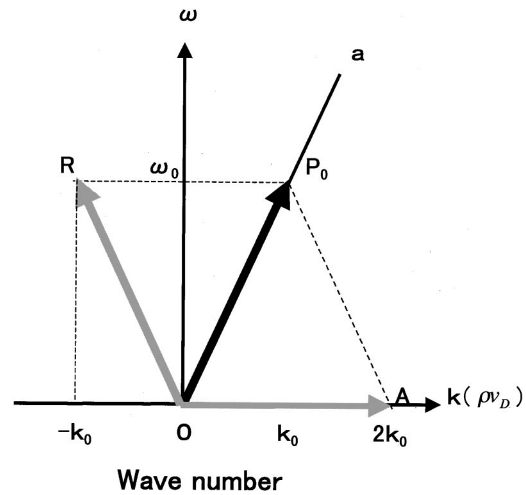

FIG. 5. ω-k diagram of acoustic wave reflection and radiation pressure.

Figure 5 shows the ω-k diagram of acoustic radiation pressure generated by the acoustic wave reflection. Vector *OA* represents acoustic radiation pressure, and vector OR is a reflected wave. In this case, there is no kinetic energy transfer.

III. DISCUSSION

We clearly distinguished between acoustic streaming and acoustic radiation pressure under ideal conditions. The distinction depends on kinetic energy transfer. In Eq. (6), kinetic energy transfer occurs completely; in Eq. (8), there is no kinetic energy transfer. However, there is an intermediate condition: the medium can partially obtain kinetic energy. This means that the medium drifts, receiving acoustic radiation pressure. These conditions are drawn as a vector from the origin to the point on line AM_D, which is represented between the horizontal line and parabolic line *O-m* in Fig. 3. It is rather difficult to estimate the kinetic energy transfer to the medium. For example, the viscosity of the medium will change the rate of the kinetic energy transfer. In an actual medium, the radiation pressure and driving force may act simultaneously. Therefore, it is rather difficult to separate acoustic radiation pressure and the driving force of acoustic streaming.

We discussed acoustic waves from the viewpoint of a quantum mechanical representation. The energy is represented by the phonon density and the frequency; therefore, there are two parameters concerning energy. This means that energy transfer is represented not only by a phonon density decrease but also by a frequency decline. According to quantum mechanics, a frequency decline indicates an energy loss. However, at the present stage, we consider acoustic streaming and acoustic radiation pressure under the condition of constant frequency. Therefore, we discuss the absorption and reflection via phonon density change. Nevertheless, we have experimentally detected the side band of the driving frequency at a large amplitude acoustic wave. In particular, a

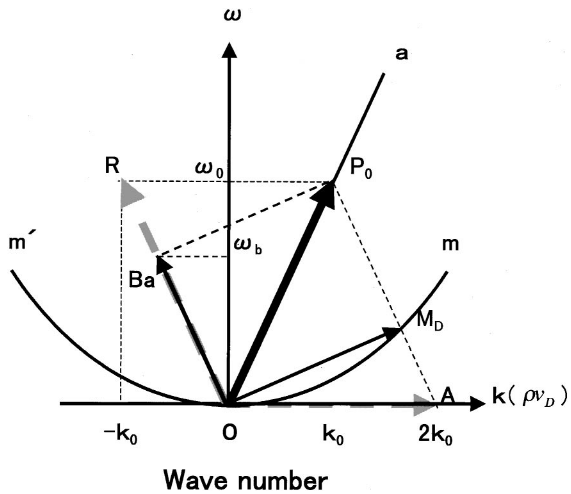

FIG. 6. ω-k diagram of acoustic wave backscattering and acoustic streaming.

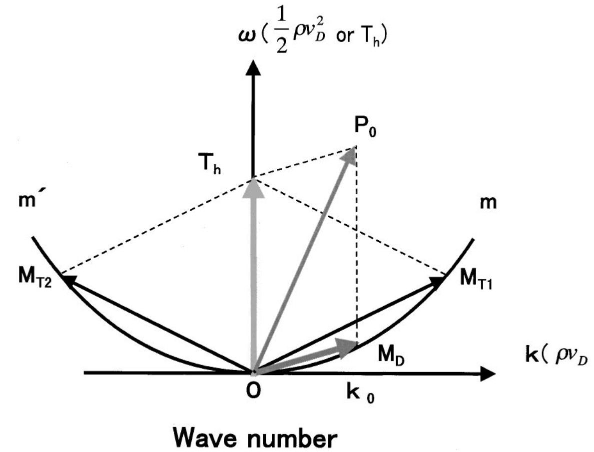

FIG. 7. ω-k diagram of thermal motion.

lower side band seems to be due to the frequency redshift of an acoustic wave [18].

In this study, we assumed the condition of constant frequency; therefore, we did not discuss the acoustic streaming caused by the backscattering of an acoustic wave. Because a kinetic energy transfer due to backscattering requires a frequency redshift of the backscattered wave, this means that energy and momentum matching conditions are not fulfilled without a frequency decline of the backscattered wave. The parallelogram cannot be drawn without the frequency decline of the backscattered wave. Figure 6 shows the ω-k diagram of backscattering and acoustic streaming. Energy and momentum matching conditions are represented as parallelogram OM_DP_0Ba. The incident acoustic wave P_0 causes medium motion OM_D and is scattered as backscattered wave OBa. Therefore the frequency of the backscattered wave is redshifted from ω_0 to ω_b. Under actual conditions, the radiation pressure and driving force may be acting simultaneously on the medium. The medium motion is represented as the vector from the origin to the point on line AM_D.

The thermal energy and kinetic energy are clearly distinguished. The thermal energy is represented as a point on the ω axis; this means that the thermal energy does not have momentum. Figure 7 shows the ω-k diagram of acoustic phonon transfer to the thermal motion and drift motion. Incident wave P_0 is represented as the summation of thermal energy O_{Th} and medium drift M_D. The thermal energy O_{Th} is also represented as the summation of two kinetic motions OM_{T1} and OM_{T2}. These kinetic motions cause the deformation of the velocity distribution function as shown in Fig. 2. Thereafter, the collision between the molecules of the medium generates a Maxwell distribution function; thus the temperature is defined as the width of the distribution function. Therefore, the averaged momentum of thermal molecular motions is 0.

IV. CONCLUSIONS

We proposed a quantum mechanical representation of acoustic waves, and investigated the interaction between an acoustic wave and a medium from the viewpoints of energy and momentum. The explanation based on the ω-k diagram is very simple, and is helpful for understanding the phenomenon compared to an analyses by hydrodynamics using the perturbation method. The simplicity of the quantum mechanical representation puts the technological application of acoustics into perspective. Acoustic wave interaction with the medium is a mixture of acoustic radiation pressure and acoustic streaming, which are caused by the absorption and reflection of the acoustic wave. At this stage, these discussions are held under the condition of constant frequency; therefore, the discussions are restricted. We plan to study the frequency change of an acoustic wave.

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